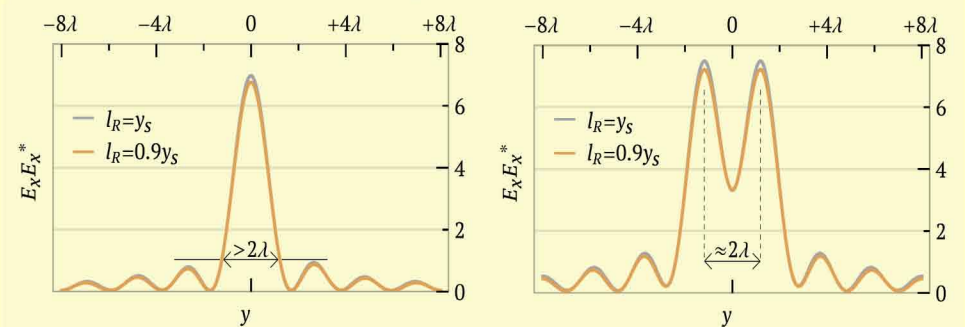
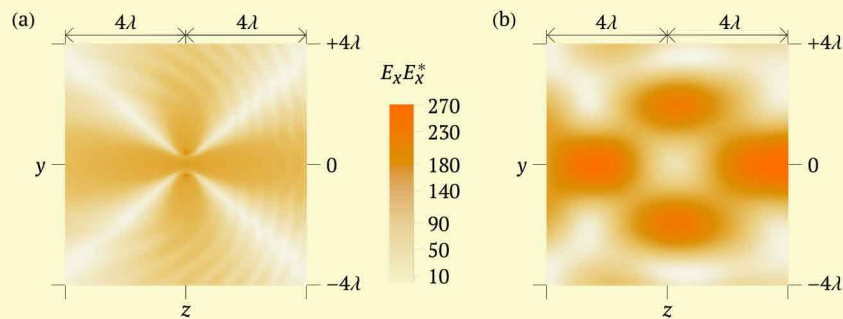
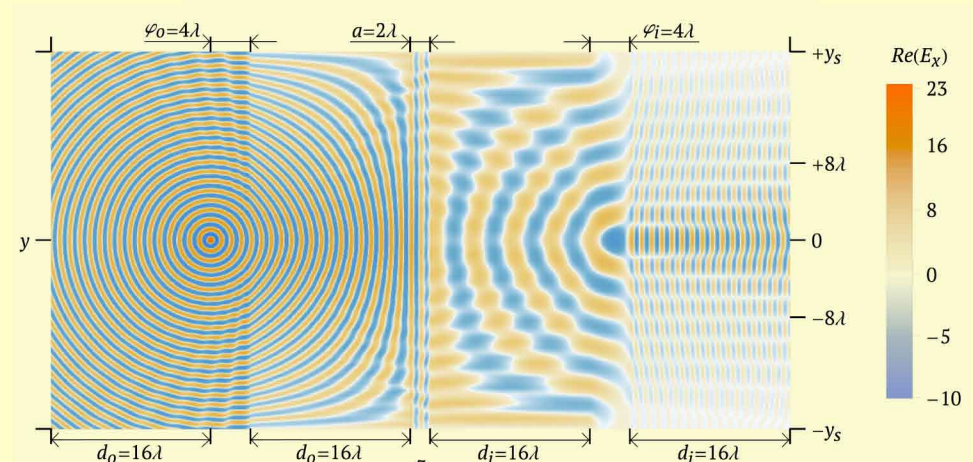
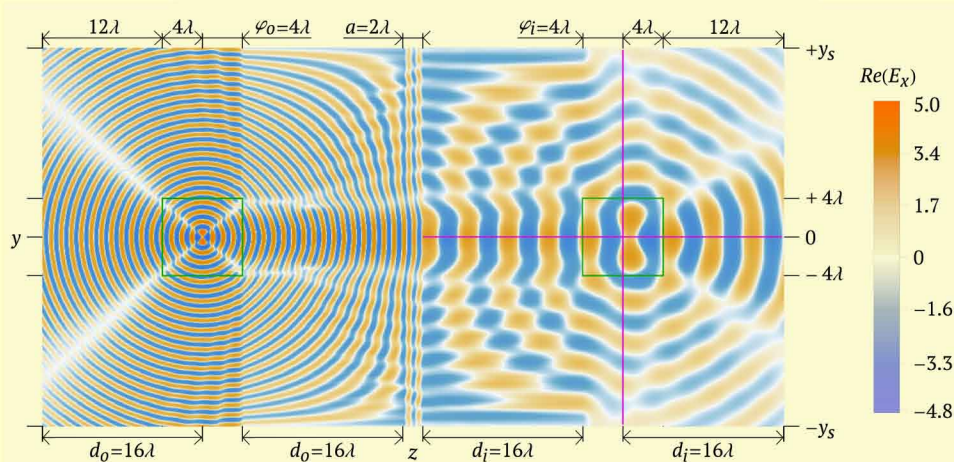
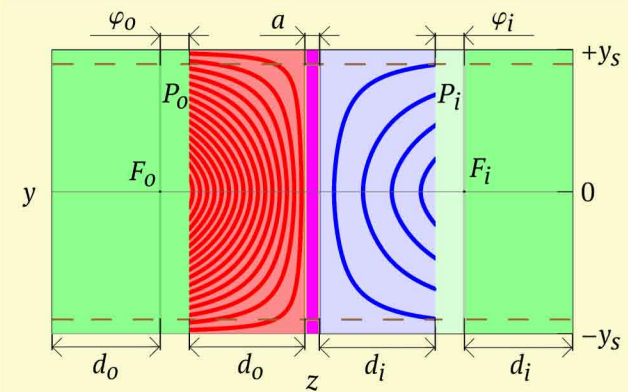
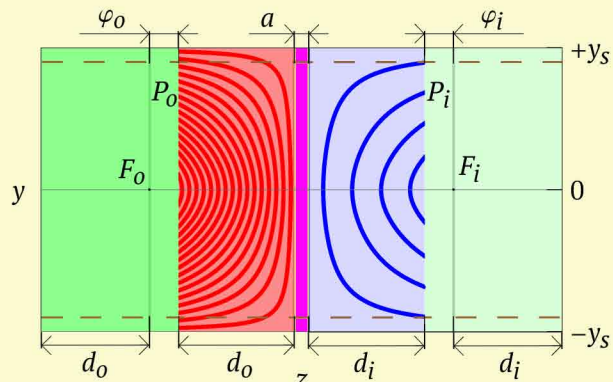


Summary

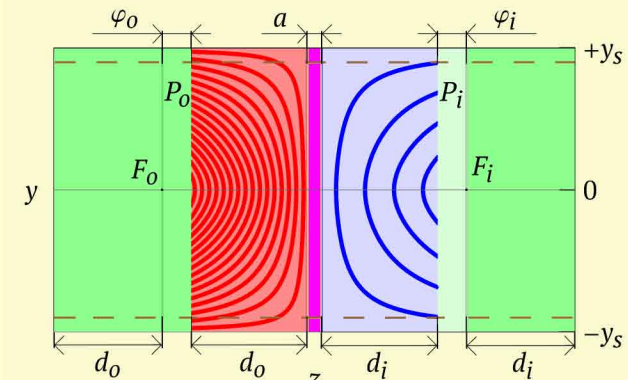
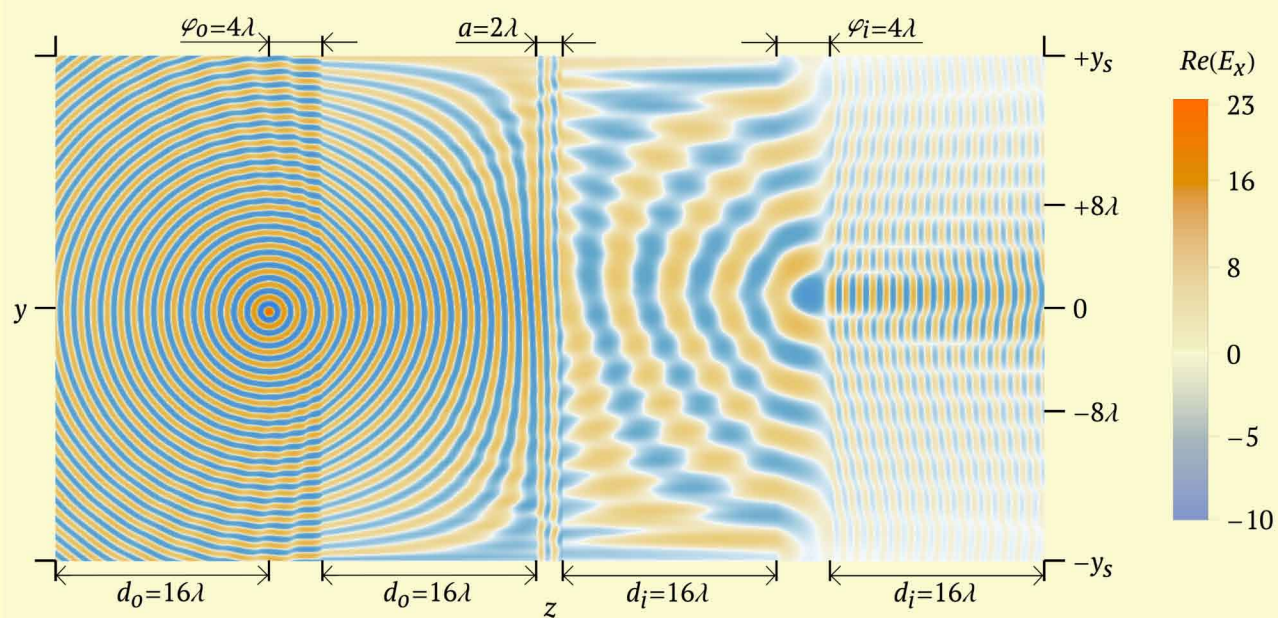
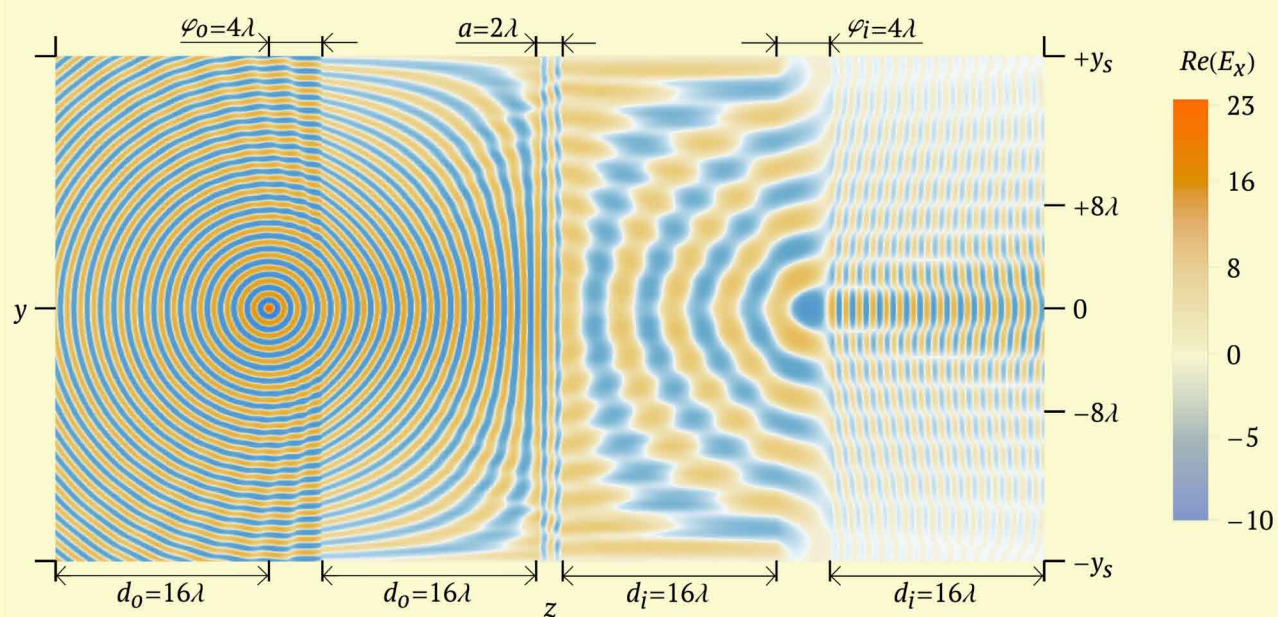


Contact information: Mircea Giloan, e-mail: mircea_giloan@yahoo.com

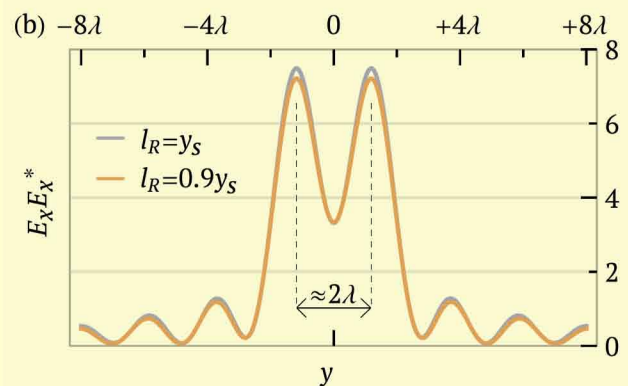
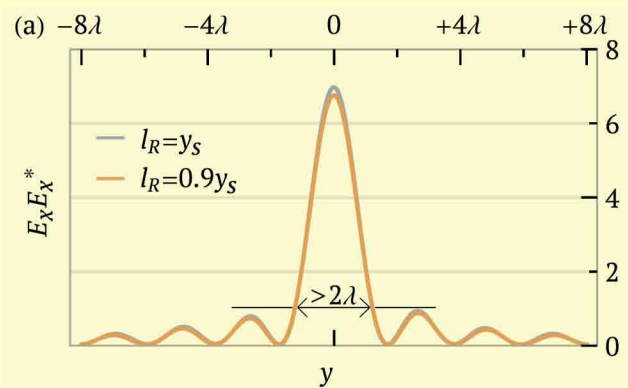
Company for Applied Informatics, Cluj-Napoca, Romania

3) Optical imaging devices based on flat lenses

2D FDTD simulations

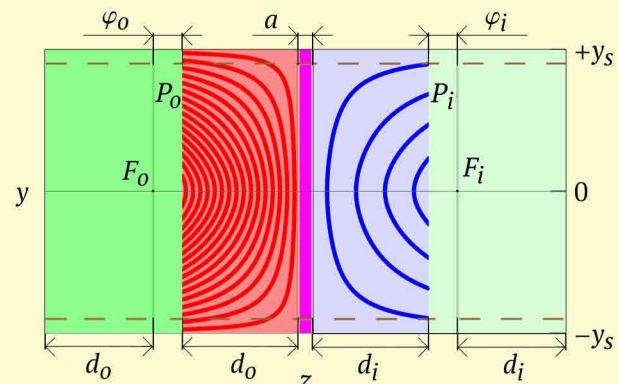
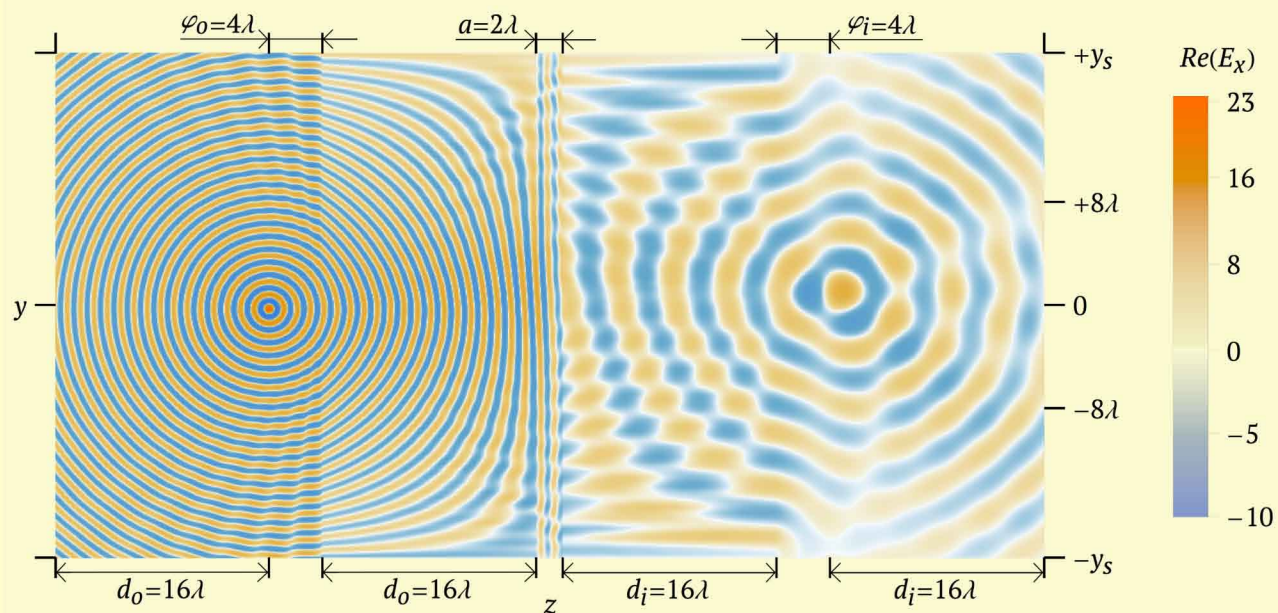
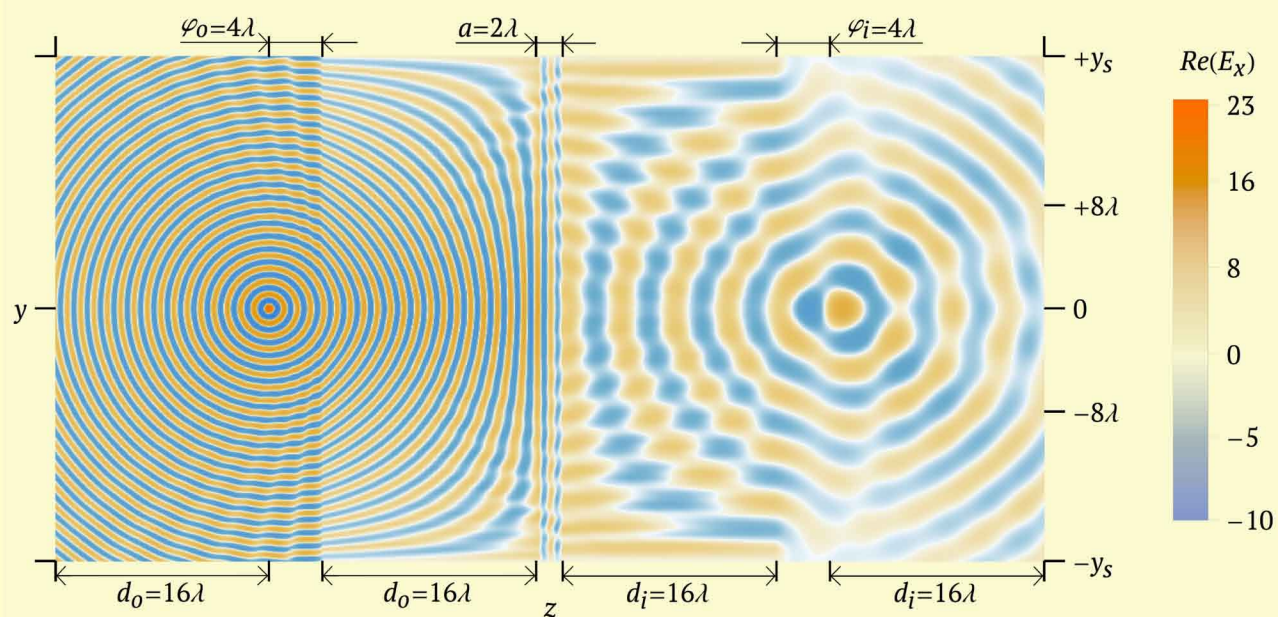


$\sigma_o = \lambda/2$ $4\times$ $\sigma_i \approx 2\lambda$

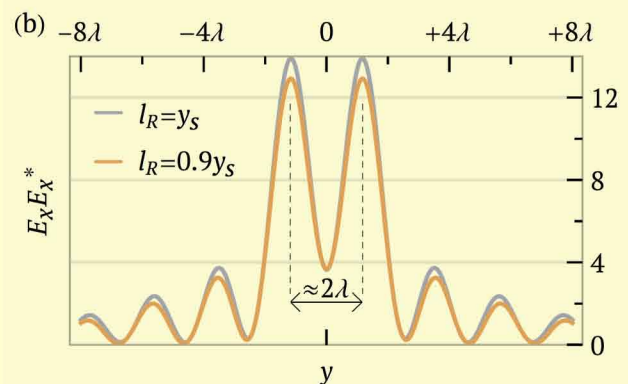
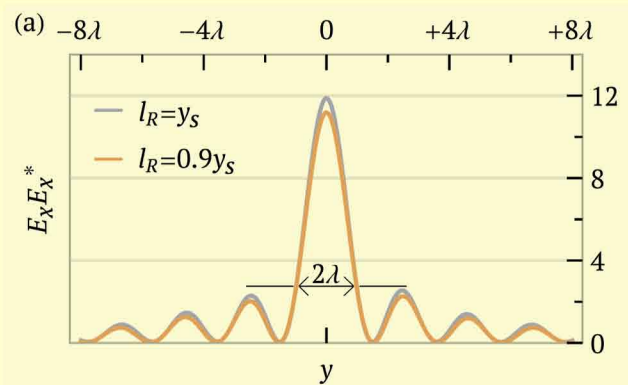


3) Optical imaging devices based on flat lenses

2D FDTD simulations

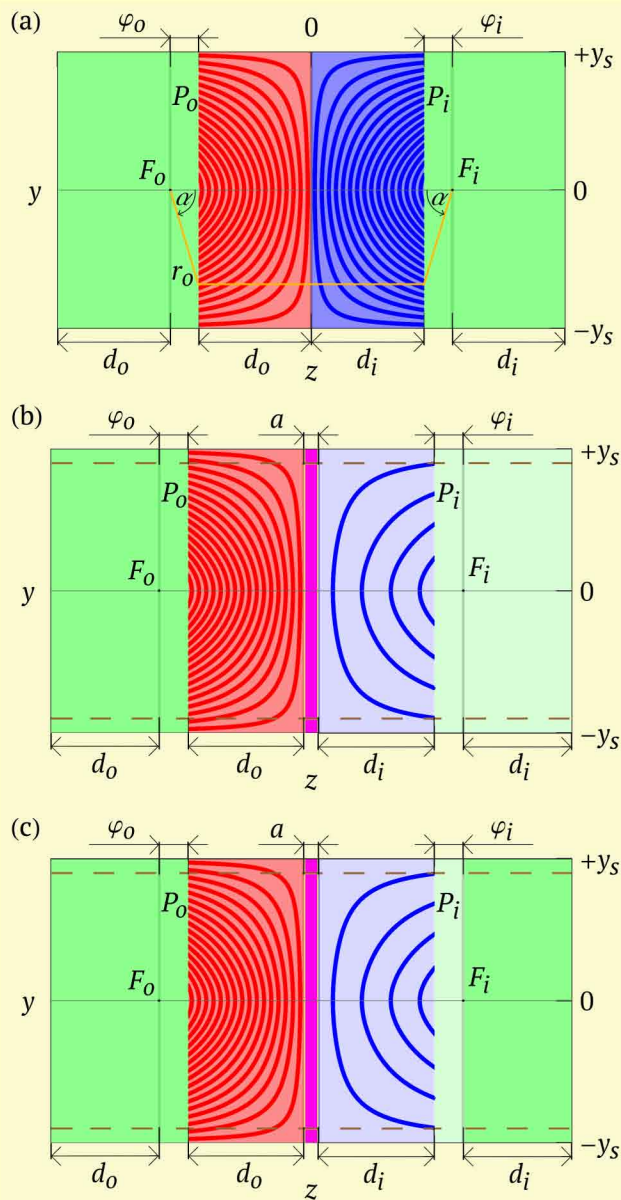


$\sigma_o = \lambda/2$ $4\times$ $\sigma_i \approx 2\lambda$



3) Optical imaging devices based on flat lenses

Optical imaging device



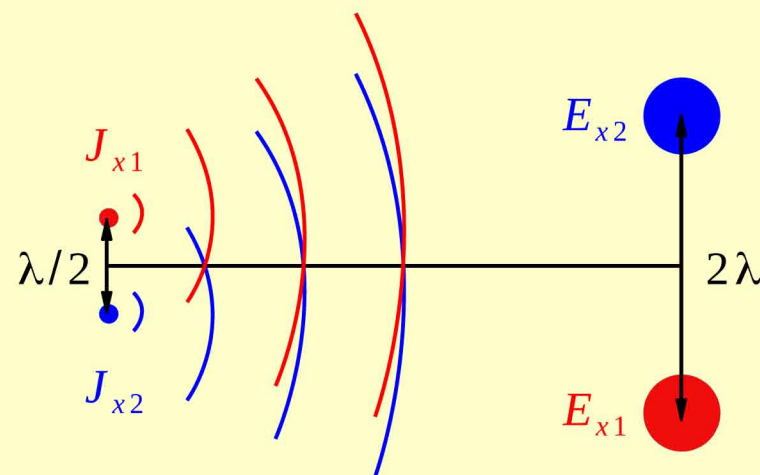
Incoherent dipole sources

$$\{E_x, H_y, H_z\}$$

$$J_x = J_0 e^{-(t-t_0)^2/2\tau^2} e^{-i\omega t + \phi}$$

$$\tau = T/(\sqrt{2})$$

$$\omega = 2\pi/T$$



$$J_{x1} = J_1 e^{-i\omega t}$$

$$J_{x2} = J_2 e^{-i\omega t + \phi}$$

$$I_1 = E_{x1} \cdot E_{x1}^*$$

$$I_2 = E_{x2} \cdot E_{x2}^*$$

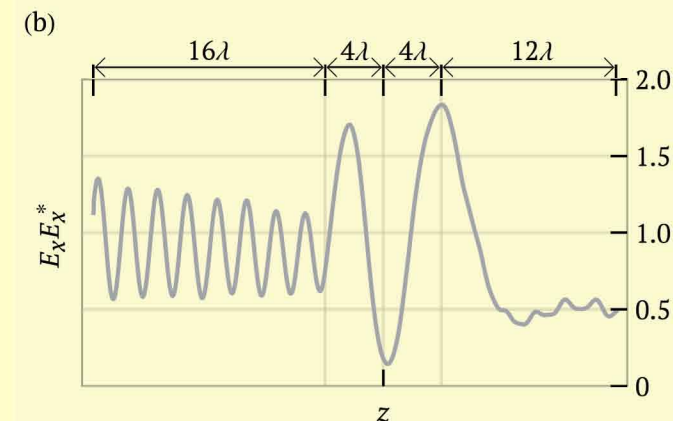
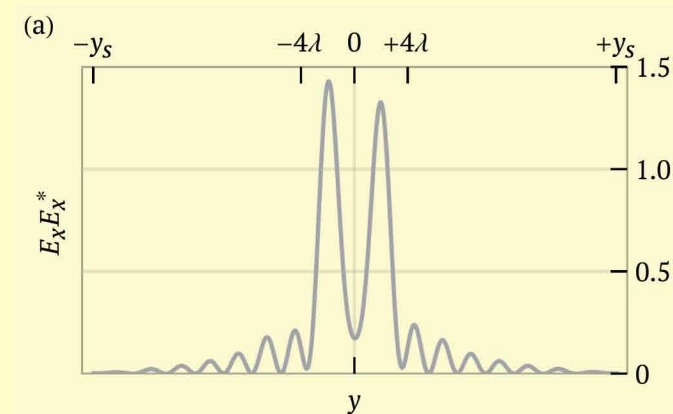
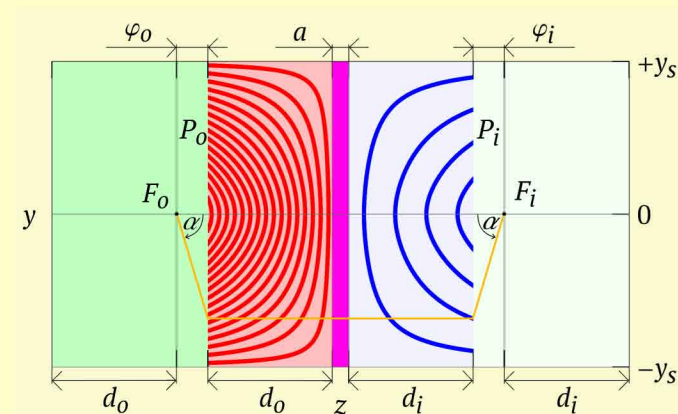
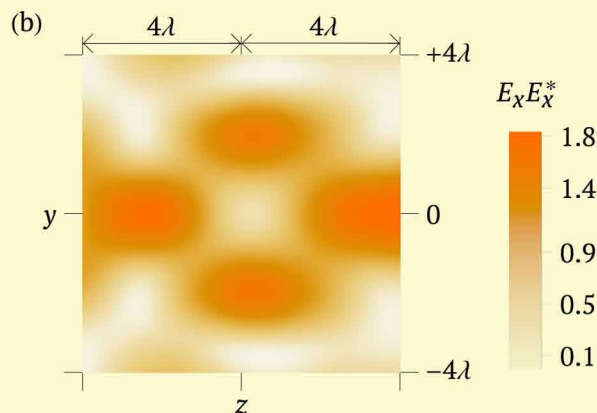
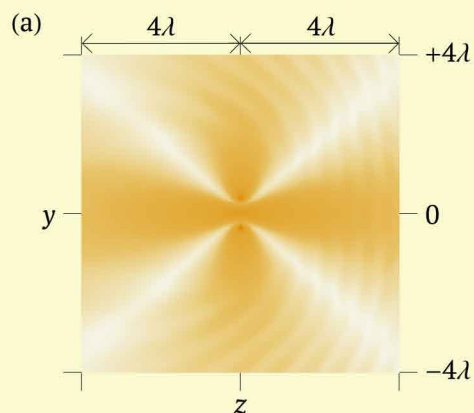
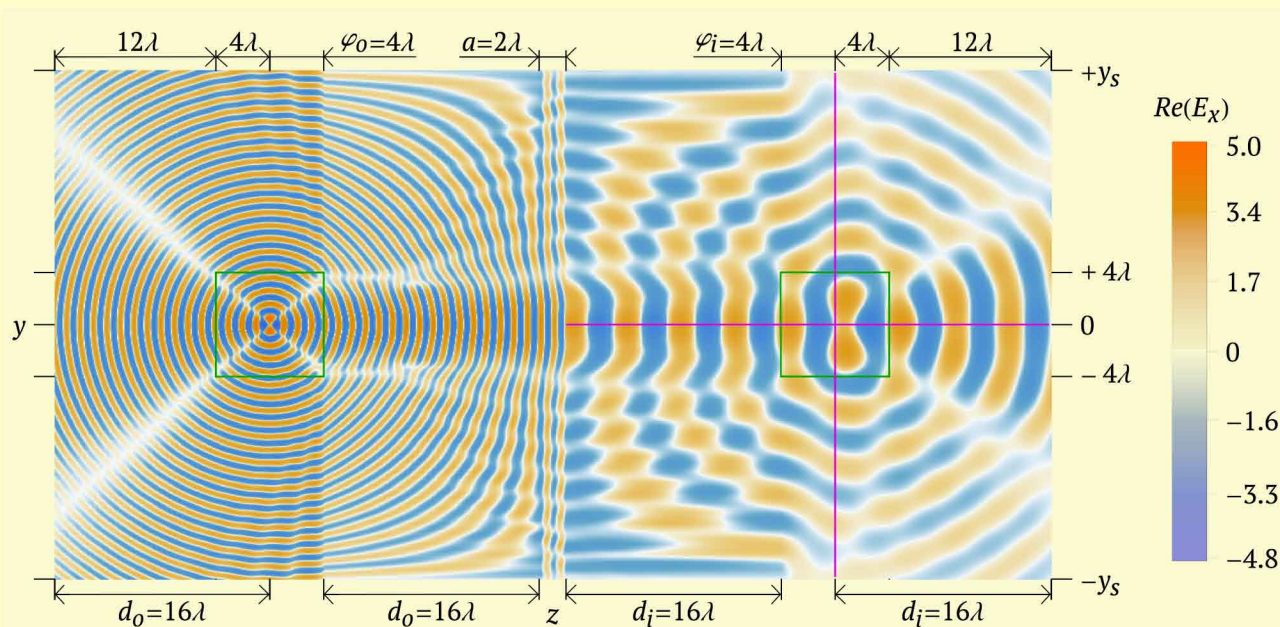
$$I = \frac{1}{2\pi} \int_0^{2\pi} (E_{x1} + E_{x2}) \cdot (E_{x1} + E_{x2})^* d\phi = I_1 + I_2$$

3) Optical imaging devices based on flat lenses

2D FDTD simulations – in phase dipole sources

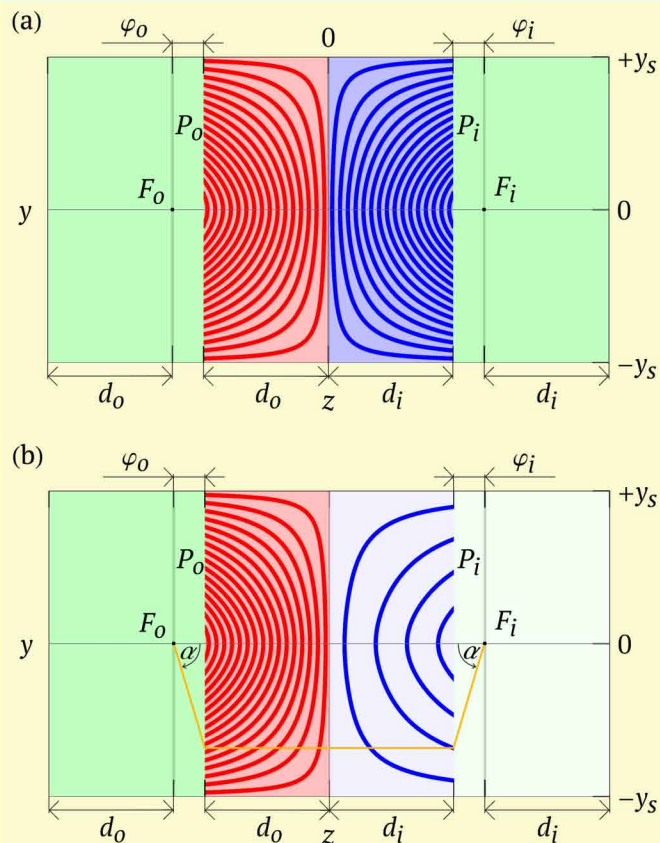
$$\{E_x, H_y, H_z\} \quad \Delta = \lambda/40 \quad \Delta t = \Delta/(c\sqrt{2}) \quad c = 1$$

$$\sigma_o = 3\lambda/4 \quad 4\times \quad \sigma_i \approx 3\lambda$$



3) Optical imaging devices based on flat lenses

Optical imaging device



$$\rho = 4$$

$$h(x, y) = m \left(\delta - \gamma \cdot \left(\varphi^2 + (x - x_F)^2 + (y - y_F)^2 \right)^{1/2} \right)$$

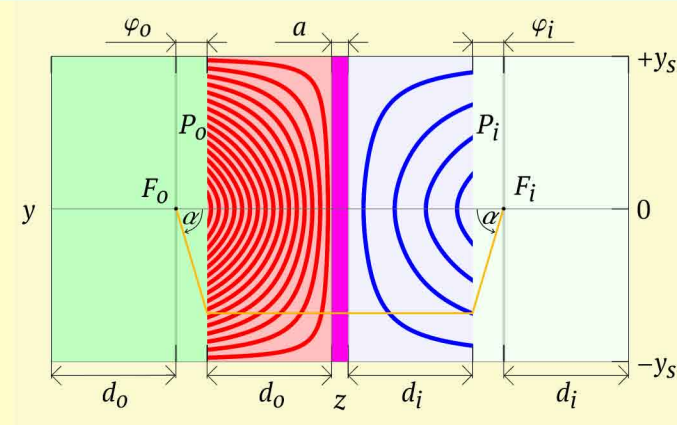
$$\delta = 1 + \varphi/d \quad \gamma = 1/d \quad \rho = d/\varphi$$

$$m_o = 1 \quad m_i = 1/4$$

$$\varepsilon = \mu = 1$$

$$\varepsilon = \mu = 1/4$$

Absorbing layer



$$t_o = \frac{2 \cos(\alpha)}{1 + \cos(\alpha)} \quad t_i = \frac{2}{1 + \cos(\alpha)}$$

$$y_s = \varphi (\rho (\rho + 2))^{1/2}$$

$$\cos(\alpha_s) = (\rho + 1)^{-1}$$

$$t_{om} = \frac{2}{\rho + 2} \quad t_{im} = \frac{2(\rho + 1)}{\rho + 2}$$

$$abs = \frac{t_{om} t_{im}}{t_o t_i}$$

Flat lens - numerical simulations

Converging lens - constitutive tensor eigenvalues

$$h(x, y) = \delta - \gamma \cdot (\varphi^2 + (x - x_F)^2 + (y - y_F)^2)^{1/2}$$

$$\gamma = +1/z_D$$

$$\varphi > 0$$

$$\delta = 1 + \varphi/z_D$$

$$(x - x_F)^2 + (y - y_F)^2 = z_D \cdot (z_D + 2\varphi)$$

$$h(x, y) \leq 0$$



$$\lambda_1 \in (-\infty, 0]$$

$$\lambda_2 \in (-\infty, -1]$$

$$\lambda_3 \in [-1, 0)$$

$$0 < h(x, y) \leq 1$$



$$\lambda_1 \in (0, 1]$$

$$\lambda_2 \in (0, 1]$$

$$\lambda_3 \in [1, \infty)$$

Mapping eigenvalues to a free loss Drude dispersive material model

$$\lambda_i(\omega) = \varepsilon_{i\infty} - \frac{\omega_{ip}^2}{\omega^2 - i\omega\gamma_i} \xrightarrow{\gamma_i=0} \lambda_i(\omega) = \frac{\omega^2 \varepsilon_{i\infty} - \omega_{ip}^2}{\omega^2}$$

$$\varepsilon_{i\infty} = \max(1, \lambda_1)$$

$$\omega_{ip}^2 = \omega_0^2 (\varepsilon_{i\infty} - \lambda_i)$$

Flat lens - numerical simulations

Finite Differences Time Domain numerical simulations

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} - \mathbf{M}$$

$$\mathbf{H} = \mu_0^{-1} \mu^{-1} \mathbf{B}$$

$$\frac{\partial \mathbf{D}}{\partial t} = \nabla \times \mathbf{H} - \mathbf{J}$$

$$\mathbf{E} = \varepsilon_0^{-1} \varepsilon^{-1} \mathbf{D}$$

$$\varepsilon = \mu = \begin{pmatrix} h & 0 & -zh_x \\ 0 & h & -zh_y \\ -zh_x & -zh_y & \frac{g^2+1}{h} \end{pmatrix}$$

$$g^2 = z^2(h_x^2 + h_y^2)$$

$$n = \begin{pmatrix} n_{xx} & 0 & n_{xz} \\ 0 & n_{yy} & n_{yz} \\ n_{xz} & n_{yz} & n_{zz} \end{pmatrix}$$

$$n_{xx} = n_{yy} = h$$

$$n_{xz} = -zh_x$$

$$n_{yz} = -zh_y$$

$$n_{zz} = \frac{g^2+1}{h}$$

Normal diagonal decomposition of permittivity and permeability tensors

$$n = D \cdot \Lambda \cdot D^T$$

$$n^{-1} = D \cdot \Lambda^{-1} \cdot D^T$$

$$D \cdot D^T = 1$$

$$\Lambda = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix}$$

$$D = \begin{pmatrix} -\rho_y & \rho_x \xi & \rho_x \beta \xi \\ \rho_x & \rho_y \xi & \rho_y \beta \xi \\ 0 & \beta \xi & -\xi \end{pmatrix}$$

$$\beta = \frac{\lambda_2 - n_{yy}}{\sqrt{n_{xz}^2 + n_{yz}^2}}$$

$$\xi = (1 + \beta^2)^{-1/2}$$

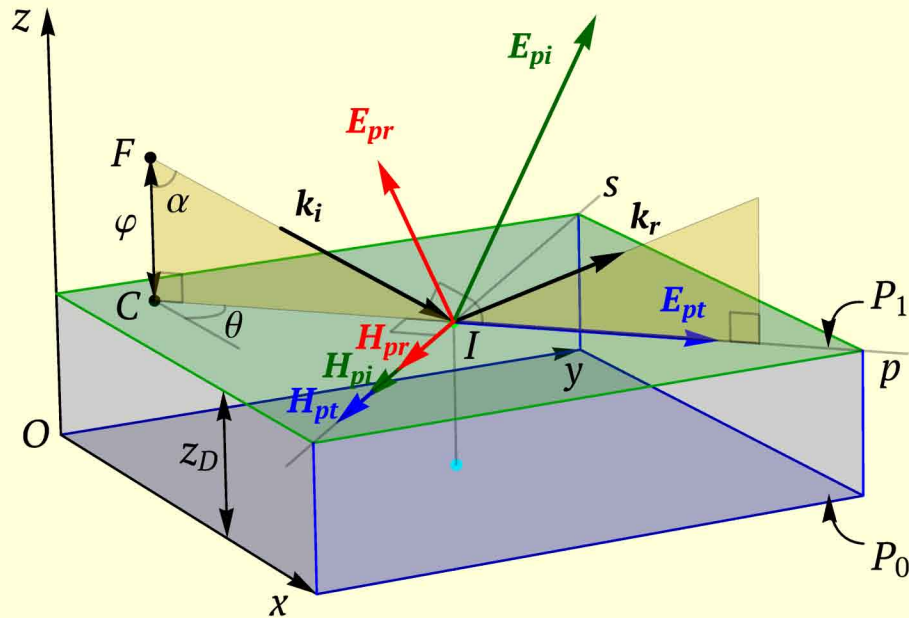
$$\lambda_1 = h \quad \lambda_{2,3} = \frac{1}{2} (n_{yy} + n_{zz} \mp \sqrt{(n_{yy} + n_{zz})^2 - 4})$$

$$\rho_x = \frac{n_{xz}}{\sqrt{n_{xz}^2 + n_{yz}^2}}$$

$$\rho_y = \frac{n_{yz}}{\sqrt{n_{xz}^2 + n_{yz}^2}}$$

2) Reflection and transmission of flat lenses media

Reflection and transmission coefficients – backward propagation



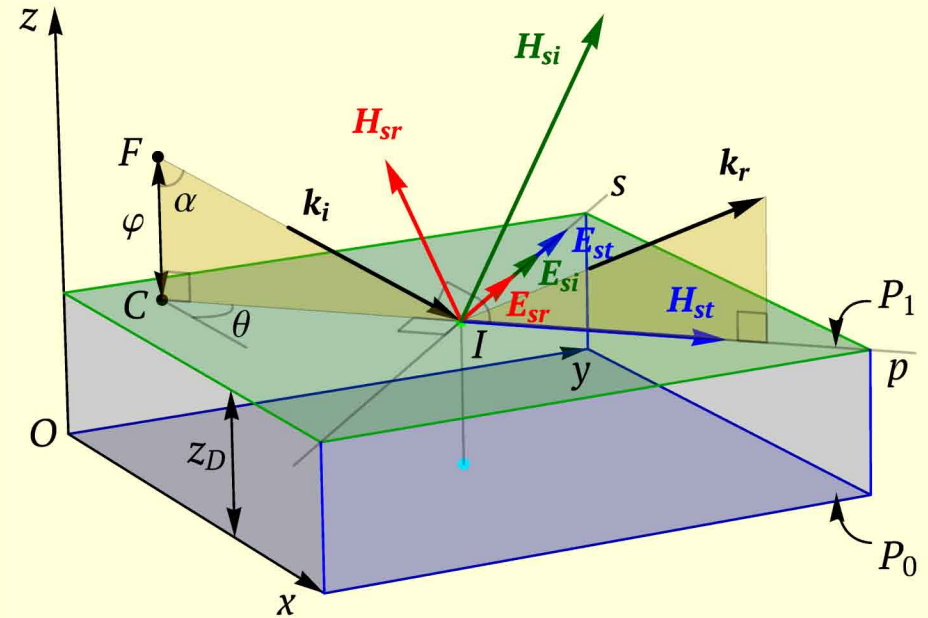
$$E_{pi} = H_{pi}$$

$$E_{pr} = H_{pr}$$

$$E_{pt} = H_{pt}$$

$$E_{pi} \cos \alpha - E_{pr} \cos \alpha = E_{pt}$$

$$H_{pi} + H_{pr} = H_{pt}$$



$$E_{si} = H_{si}$$

$$E_{sr} = H_{sr}$$

$$E_{st} = H_{st}$$

$$E_{si} + E_{sr} = E_{st}$$

$$H_{si} \cos \alpha - H_{sr} \cos \alpha = H_{st}$$

$$r = \frac{E_{pr}}{E_{pi}} = \frac{E_{sr}}{E_{si}} = -\frac{1 - \cos \alpha}{1 + \cos \alpha}$$

$$t = \frac{E_{pt}}{E_{pi}} = \frac{E_{st}}{E_{si}} = \frac{2 \cos \alpha}{1 + \cos \alpha}$$

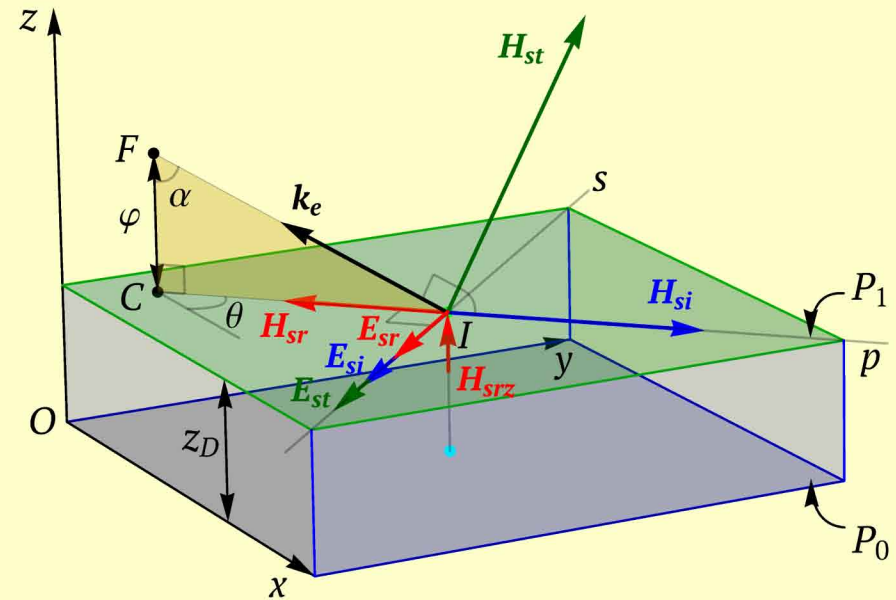
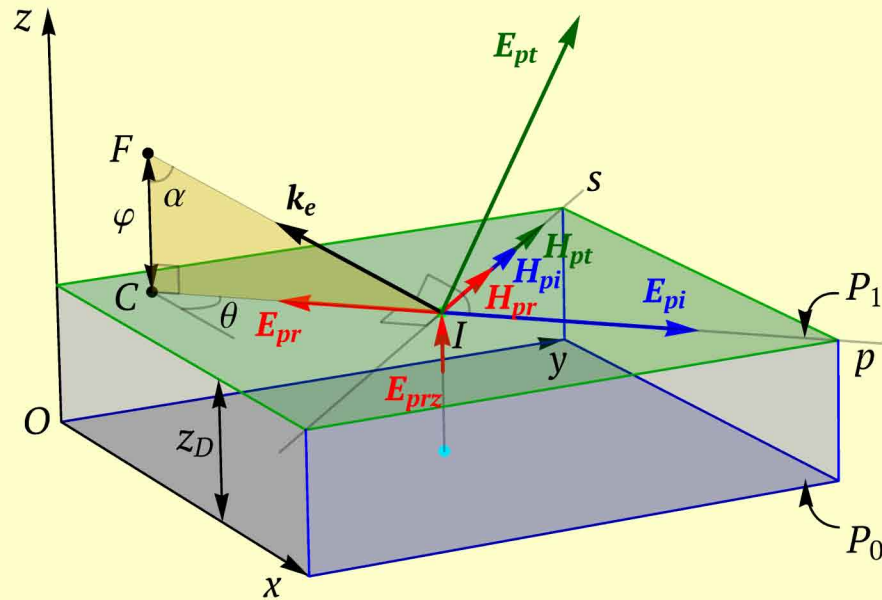
$$R = r^2$$

$$T = t^2 / \cos \alpha$$

$$R + T = 1$$

2) Reflection and transmission of flat lenses media

Reflection and transmission coefficients – forward propagation



$$\begin{aligned} E_{pi} &= H_{pi} \\ E_{pr} &= H_{pr} \\ E_{pt} &= H_{pt} \end{aligned}$$

$$\begin{aligned} E_{pi} - E_{pr} &= E_{pt} \cos \alpha \\ H_{pi} + H_{pr} &= H_{pt} \end{aligned}$$

$$\begin{aligned} E_{si} &= H_{si} \\ E_{sr} &= H_{sr} \\ E_{st} &= H_{st} \end{aligned}$$

$$\begin{aligned} E_{si} + E_{sr} &= E_{st} \\ H_{si} - H_{sr} &= H_{st} \cos \alpha \end{aligned}$$

$$\begin{aligned} r &= \frac{E_{pr}}{E_{pi}} = \frac{E_{sr}}{E_{si}} = \frac{1 - \cos \alpha}{1 + \cos \alpha} \\ t &= \frac{E_{pt}}{E_{pi}} = \frac{E_{st}}{E_{si}} = \frac{2}{1 + \cos \alpha} \end{aligned}$$

$$R = r^2$$

$$T = t^2 \cos \alpha$$

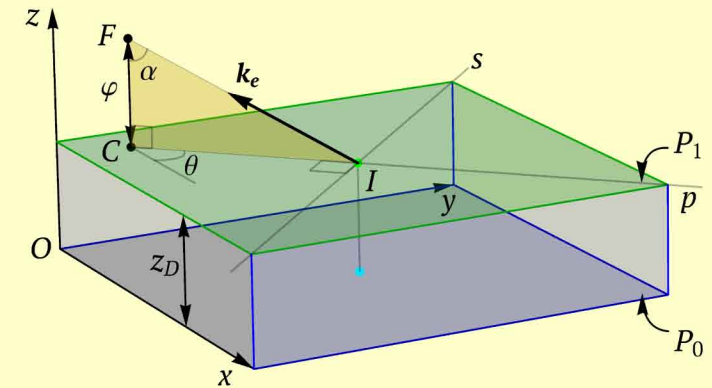
$$R + T = 1$$

2) Reflection and transmission of flat lenses media

Incident wave at the output interface of the lens – forward propagation

$$M_i = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/h \end{pmatrix} \xrightarrow{M_i \cdot \mathbf{E}_i = 0} \mathbf{E}_i \begin{pmatrix} E_{xi} \\ E_{yi} \\ 0 \end{pmatrix}$$

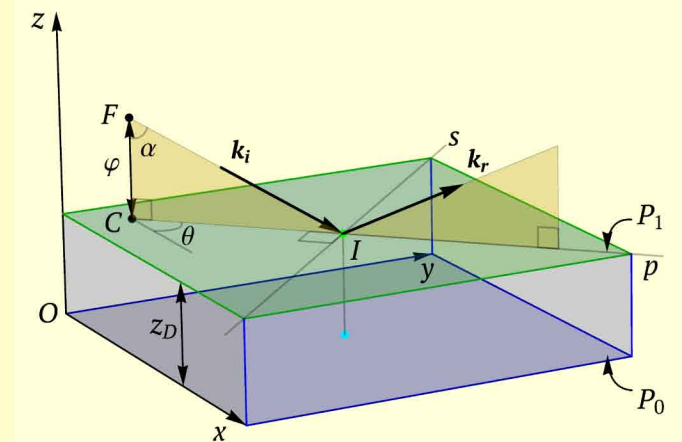
$$P_i = \begin{pmatrix} 0 & -1 & z_D \cdot h_y / h \\ 1 & 0 & -z_D \cdot h_x / h \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{P_i \cdot \mathbf{E}_i = \mathbf{H}_i} \mathbf{H}_i \begin{pmatrix} H_{xi} \\ H_{yi} \\ 0 \end{pmatrix}$$



Transmitted wave at the output interface of the lens – backward propagation

$$M_t = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/h \end{pmatrix} \xrightarrow{M_t \cdot \mathbf{E}_t = 0} \mathbf{E}_t \begin{pmatrix} E_{xt} \\ E_{yt} \\ 0 \end{pmatrix}$$

$$P_t = \begin{pmatrix} 0 & 1 & -z_D \cdot h_y / h \\ -1 & 0 & z_D \cdot h_x / h \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{P_t \cdot \mathbf{E}_t = \mathbf{H}_t} \mathbf{H}_t \begin{pmatrix} H_{xt} \\ H_{yt} \\ 0 \end{pmatrix}$$



2) Reflection and transmission of flat lenses media

Reflected wave at the output interface of the lens – forward propagation

$$M_r = \begin{pmatrix} \frac{4z_D^2 h h_x^2}{(g_D^2+1)^2} & \frac{4z_D^2 h h_x h_y}{(g_D^2+1)^2} & \frac{-2z_D h_x}{g_D^2+1} \\ \frac{4z_D^2 h h_x h_y}{(g_D^2+1)^2} & \frac{4z_D^2 h h_y^2}{(g_D^2+1)^2} & \frac{-2z_D h_y}{g_D^2+1} \\ \frac{-2z_D h_x}{g_D^2+1} & \frac{-2z_D h_y}{g_D^2+1} & \frac{1}{h} \end{pmatrix}$$

$$M_r \cdot \mathbf{E}_r = 0$$

$$\mathbf{E}_{pr} = \begin{pmatrix} -E_{pr} \cos \theta \\ -E_{pr} \sin \theta \\ E_{pr} h \frac{2g_D}{g_D^2+1} \end{pmatrix}$$

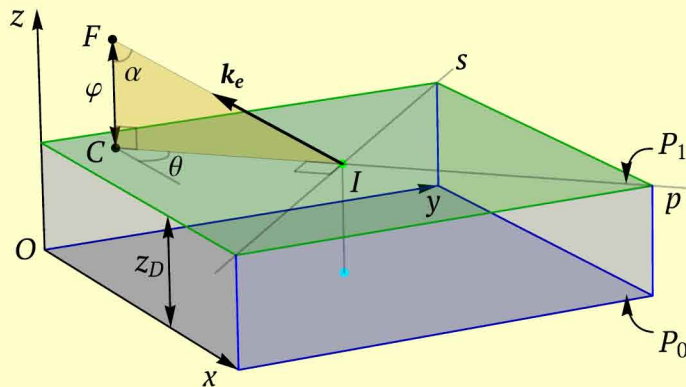
$$\mathbf{E}_{sr} = \begin{pmatrix} E_{sr} \sin \theta \\ -E_{sr} \cos \theta \\ 0 \end{pmatrix}$$

$$P_r = \begin{pmatrix} \frac{-2z_D^2 h_x h_y}{g_D^2+1} & \frac{z_D^2 (h_x^2 - h_y^2) + 1}{g_D^2+1} & \frac{z_D h_y}{h} \\ \frac{z_D^2 (h_x^2 - h_y^2) - 1}{g_D^2+1} & \frac{2z_D^2 h_x h_y}{g_D^2+1} & \frac{-z_D h_x}{h} \\ \frac{-2z_D h h_y}{g_D^2+1} & \frac{2z_D h h_x}{g_D^2+1} & 0 \end{pmatrix}$$

$$P_r \cdot \mathbf{E}_r = \mathbf{H}_r$$

$$\mathbf{H}_{pr} = \begin{pmatrix} -H_{pr} \sin \theta \\ H_{pr} \cos \theta \\ 0 \end{pmatrix}$$

$$\mathbf{H}_{sr} = \begin{pmatrix} -H_{sr} \cos \theta \\ -H_{sr} \sin \theta \\ H_{sr} h \frac{2g_D}{g_D^2+1} \end{pmatrix}$$



$$h(x, y) = \delta - \gamma \cdot (\varphi^2 + (x - x_F)^2 + (y - y_F)^2)^{1/2}$$

$$h_x \cos \theta + h_y \sin \theta = -\gamma g_D$$

$$h_x \sin \theta = h_y \cos \theta$$

$$\sin \alpha = g_D$$

2) Reflection and transmission of flat lenses media

Electromagnetic waves polarization

$$\mathbf{E} = \mathbf{E}_0 e^{i(k_0 \mathbf{k} \cdot \mathbf{r} - \omega t)}$$

$$\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$$

$$\mathbf{k} \times \mathbf{E}_0 - \mu \mathbf{H}_0 = 0$$

$$\mathbf{H} = \frac{1}{\eta_0} \mathbf{H}_0 e^{i(k_0 \mathbf{k} \cdot \mathbf{r} - \omega t)}$$

$$\mathbf{B} = \mu_0 \mu \mathbf{H}$$

$$\mathbf{k} \times [\varepsilon^{-1} (\mathbf{k} \times \mathbf{H}_0)] + \mu \mathbf{H}_0 = 0$$

$$k_0 = \omega / c = \omega \sqrt{\varepsilon_0 \mu_0}$$

$$\nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}$$

$$\mathbf{k} \times \mathbf{H}_0 + \varepsilon \mathbf{E}_0 = 0$$

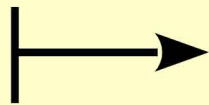
$$\eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}}$$

$$\mathbf{D} = \varepsilon_0 \varepsilon \mathbf{E}$$

$$\mathbf{k} \times [\mu^{-1} (\mathbf{k} \times \mathbf{E}_0)] + \varepsilon \mathbf{E}_0 = 0$$

Transformation media

$$\mathbf{k}^T = (k_x, k_y, k_z)$$



$$\mathbf{k} \times \mathbf{F} = \mathbf{K} \cdot \mathbf{F}$$

$$\mathbf{P} = n^{-1} \cdot \mathbf{K}$$

$$\mathbf{P} \cdot \mathbf{E}_0 = \mathbf{H}_0$$

$$\mathbf{M} \cdot \mathbf{H}_0 = 0$$

$$\mathbf{K} = \begin{pmatrix} 0 & -k_z & k_y \\ k_z & 0 & -k_x \\ -k_y & k_x & 0 \end{pmatrix}$$

$$\varepsilon = \mu = n$$

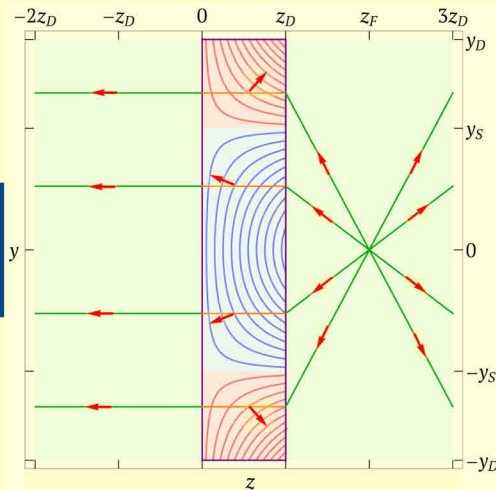
$$\mathbf{M} = \mathbf{K} \cdot n^{-1} \cdot \mathbf{K} + n$$

$$\mathbf{P} \cdot \mathbf{H}_0 = -\mathbf{E}_0$$

$$\mathbf{M} \cdot \mathbf{E}_0 = 0$$

2) Reflection and transmission of flat lenses media

Wave vectors of the incident and reflected waves at the output interface of the lens



backward propagation

from focal point

$$z = z_D, \quad k_x = -z_D h_x, \quad k_y = -z_D h_y$$



$$H(x, y, k_z) = (k_z + h)(k_z(g_D^2 + 1) + h(g_D^2 - 1))/h$$



$$H(x, y, k_z) = 0$$



$$k_{zi} = -h$$

$$P_1: z = z_D$$

$$\mathbf{k}_t \begin{pmatrix} k_{xt} = -z_D h_x \\ k_{yt} = -z_D h_y \\ k_{zt} = -h \end{pmatrix}$$

$$\mathbf{k}_i \begin{pmatrix} k_{xi} = -z_D h_x \\ k_{yi} = -z_D h_y \\ k_{zi} \end{pmatrix}$$

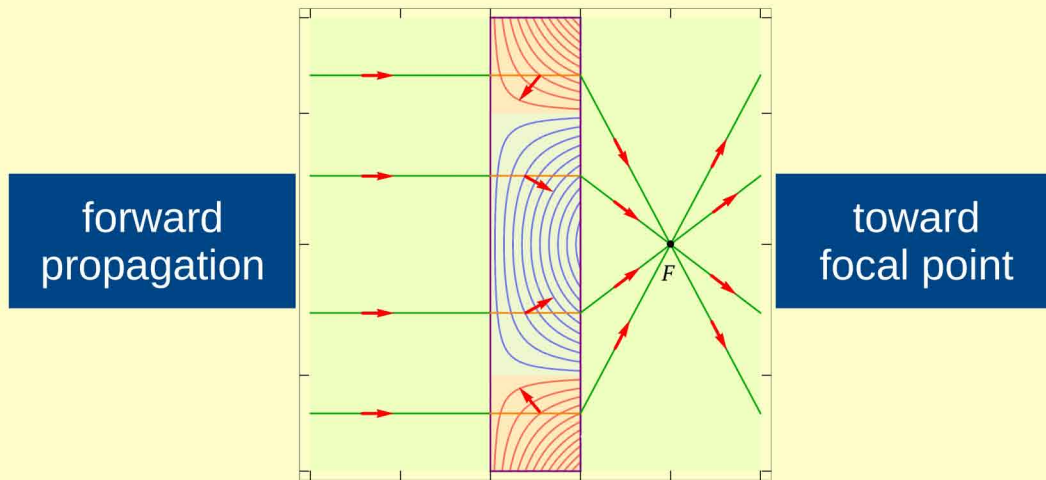
$$k_{xi}^2 + k_{yi}^2 + k_{zi}^2 = 1$$

$$\mathbf{k}_r \begin{pmatrix} k_{xr} = -z_D h_x \\ k_{yr} = -z_D h_y \\ k_{zr} \end{pmatrix}$$

$$k_{xr}^2 + k_{yr}^2 + k_{zr}^2 = 1$$

2) Reflection and transmission of flat lenses media

Wave vectors of the incident and reflected waves at the output interface of the lens



$P_1: z = z_D$

$$\mathbf{k}_i \begin{pmatrix} k_{xi} = z_D h_x \\ k_{yi} = z_D h_y \\ k_{zi} = h \end{pmatrix}$$

$$\mathbf{k}_r \begin{pmatrix} k_{xr} = z_D h_x \\ k_{yr} = z_D h_y \\ k_{zr} = h \frac{g_D^2 - 1}{g_D^2 + 1} \end{pmatrix}$$

$$\mathbf{k}_t \begin{pmatrix} k_{xt} = z_D h_x \\ k_{yt} = z_D h_y \\ k_{zt} \end{pmatrix}$$

$$k_{xt}^2 + k_{yt}^2 + k_{zt}^2 = 1$$

$$z = z_D, \quad k_x = z_D h_x, \quad k_y = z_D h_y$$



$$H(x, y, k_z) = (k_z - h)(k_z(g_D^2 + 1) - h(g_D^2 - 1))/h$$



$$H(x, y, k_z) = 0$$

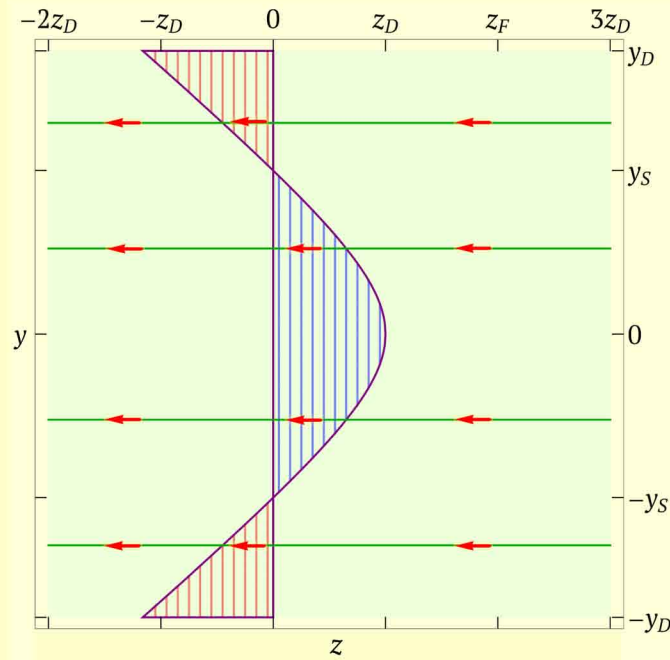


$$k_{zi} = h \quad k_{zr} = h(g_D^2 - 1)/(g_D^2 + 1)$$

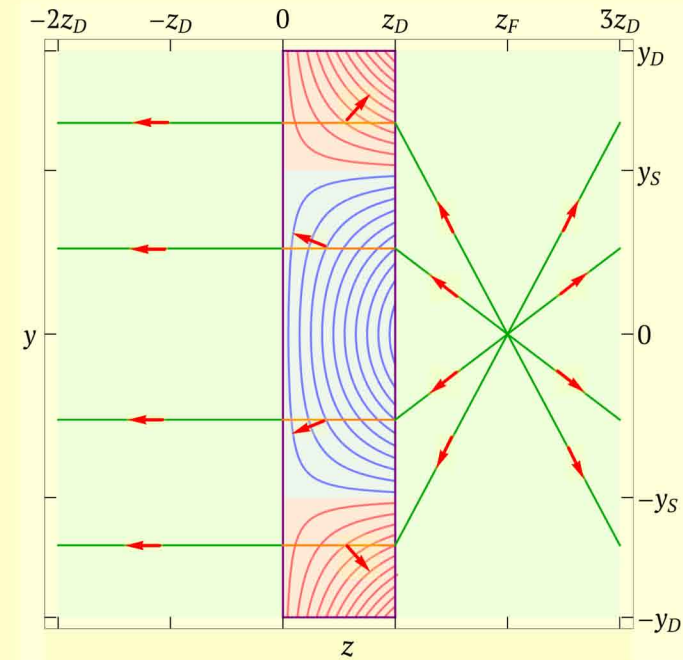
$$g_D^2 = z_D^2 (h_x^2 + h_y^2)$$

1) Flat lenses designed by transformation-optics

Converging lens – backward propagation (from focal point)



TO
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$$h(x, y) = \delta - \gamma \cdot \left(\varphi^2 + (x - x_F)^2 + (y - y_F)^2 \right)^{1/2}$$

$$z \rightarrow z/h(x, y)$$

$$\gamma = +1/z_D \quad \varphi = z_D \quad \delta = 2$$

$$r|_{z=0} = 0$$

$$r|_{z=z_D} \neq 0$$

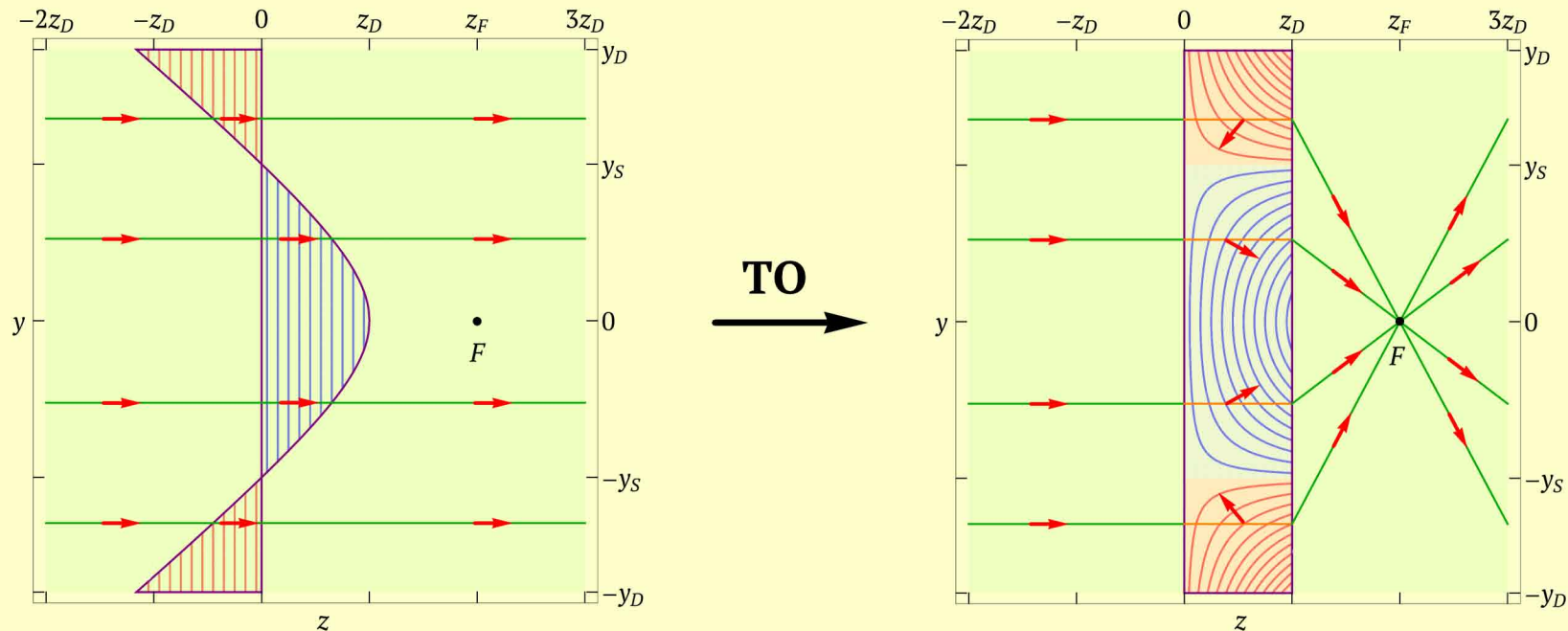
$$(x_S - x_F)^2 + (y_S - y_F)^2 = z_D \cdot (z_D + 2\varphi)$$

$$t|_{z=0} = 1$$

$$t|_{z=z_D} \neq 1$$

1) Flat lenses designed by transformation-optics

Converging lens – forward propagation (toward focal point)



$$h(x, y) = \delta - \gamma \cdot (\varphi^2 + (x - x_F)^2 + (y - y_F)^2)^{1/2}$$

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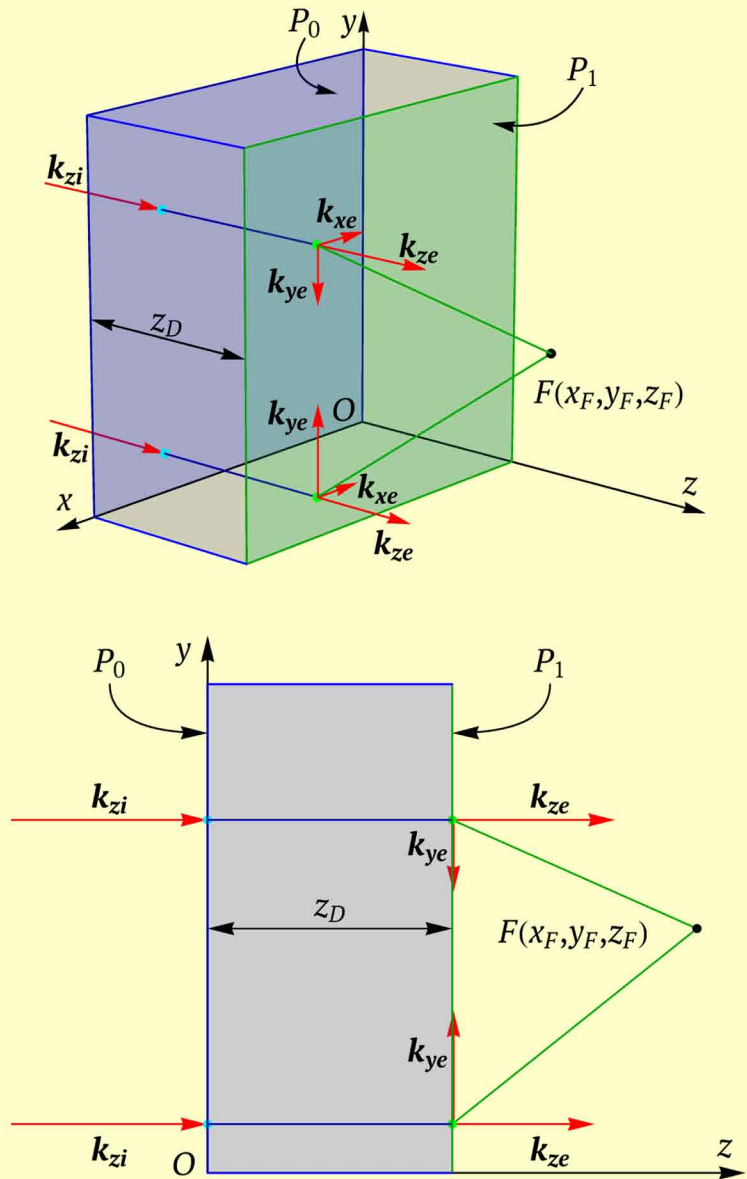
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1) Flat lenses designed by transformation-optics

Wave vector manipulation



Transformation function retrieval

$$\frac{k_{xe}}{k_{ze}} = \frac{x - x_F}{z_D - z_F}$$

$$k_{xe} = z_D \cdot h_x$$

$$k_{ye} = z_D \cdot h_y$$

$$\frac{k_{ye}}{k_{ze}} = \frac{y - y_F}{z_D - z_F}$$

$$k_{xe}^2 + k_{ye}^2 + k_{ze}^2 = 1$$

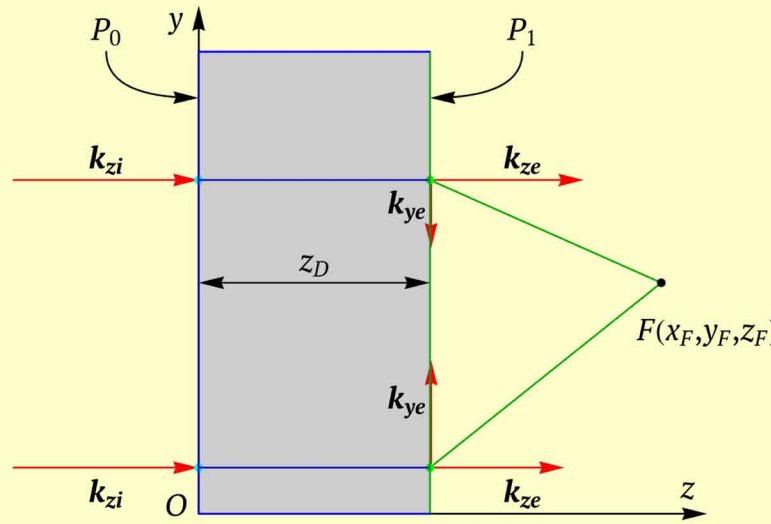
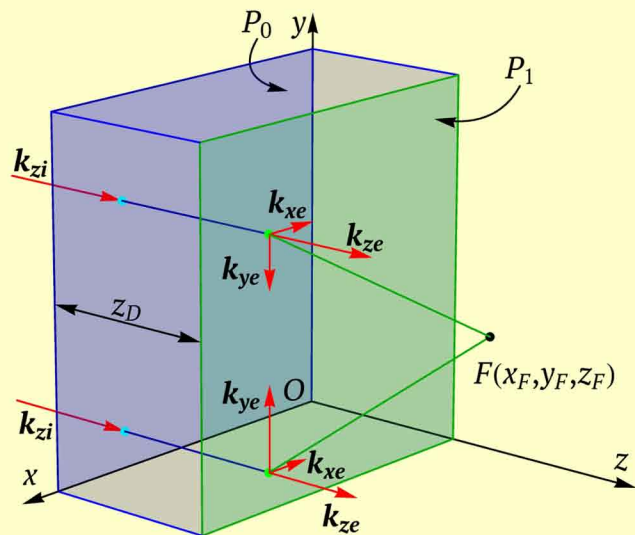
$$h_x^2 + h_y^2 = \frac{1}{z_D^2} \cdot \frac{(x - x_F)^2 + (y - y_F)^2}{\varphi^2 + (x - x_F)^2 + (y - y_F)^2}$$

$$h(x, y) = \frac{1}{f(x, y)} = \delta - \gamma \cdot \left(\varphi^2 + (x - x_F)^2 + (y - y_F)^2 \right)^{1/2}$$

$$\gamma = \pm 1/z_D \quad f(x_F, y_F) = 1 \quad \longrightarrow \quad \delta = 1 + \varphi/z_D$$

1) Flat lenses designed by transformation-optics

Devices for wave vector manipulation



$$\left\{ \begin{array}{l} x \rightarrow x \\ y \rightarrow y \\ z \rightarrow z \cdot f(x, y) \end{array} \right\}$$

$$h(x, y) = 1/f(x, y)$$

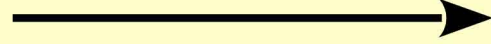
$$\left\{ \begin{array}{l} x \rightarrow x \\ y \rightarrow y \\ z \rightarrow z/h(x, y) \end{array} \right\}$$

Emergent wave at output interface ($P_1: z=z_D$)

$$k_{x1} = z_D \cdot h_x$$

$$k_{y1} = z_D \cdot h_y$$

$$k_{z1} = h$$



$$k_{xe}^2 + k_{ye}^2 + k_{ze}^2 = 1 \longrightarrow$$

$$k_{xe} = z_D \cdot h_x$$

$$k_{ye} = z_D \cdot h_y$$

$$k_{ze}$$

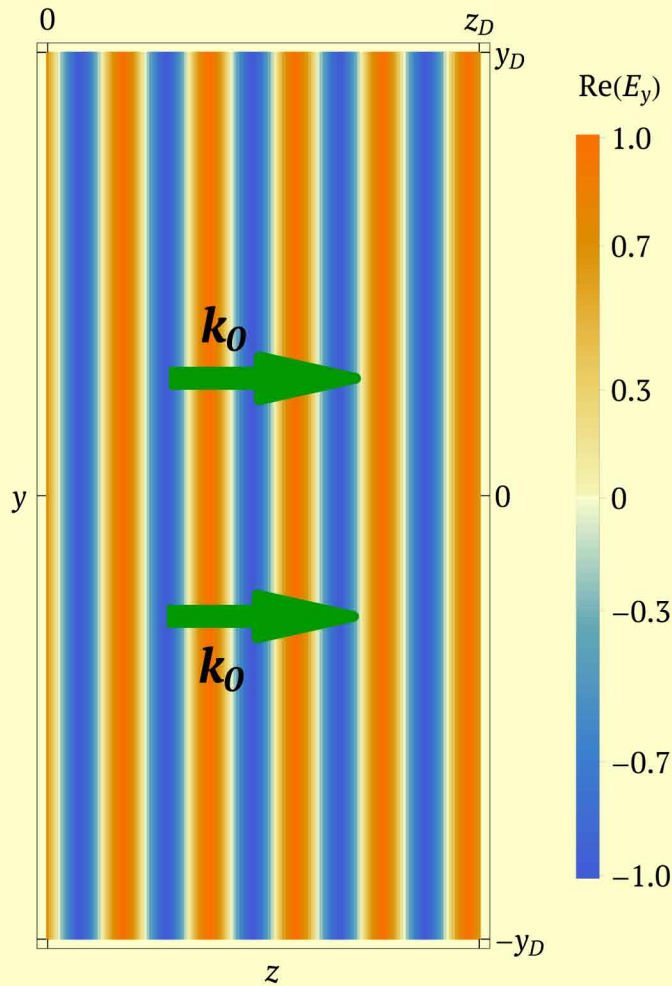
Constitutive parameters

$$\epsilon = \mu = \begin{pmatrix} h & 0 & -z h_x \\ 0 & h & -z h_y \\ -z h_x & -z h_y & (g^2 + 1)/h \end{pmatrix}$$

$$g^2 = z^2 (h_x^2 + h_y^2)$$

1) Flat lenses designed by transformation-optics

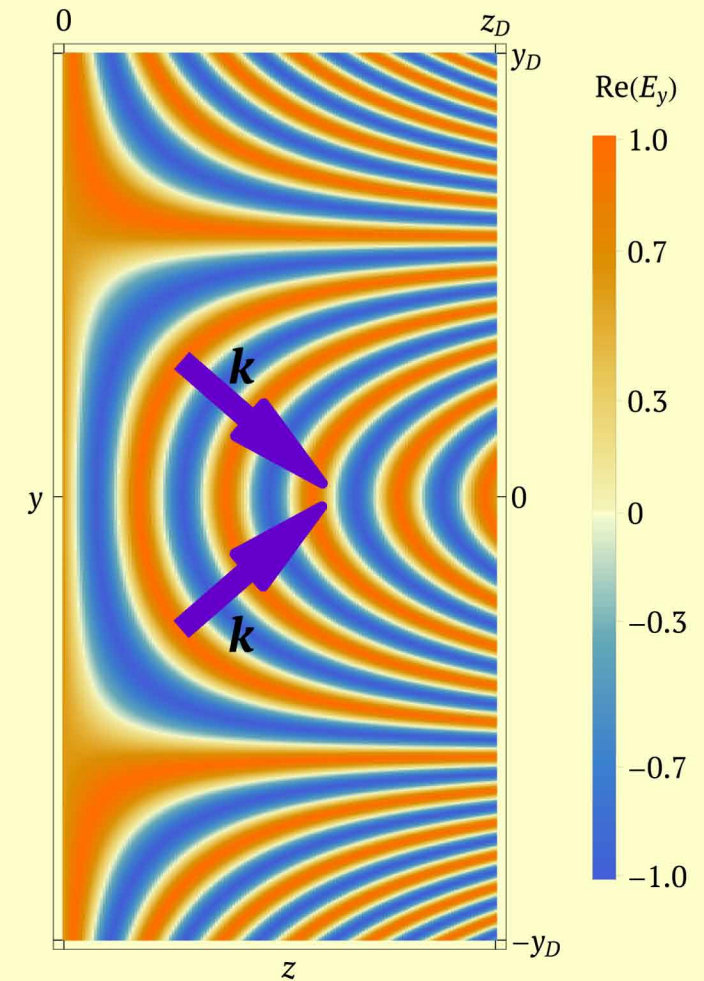
Initial space and field



Transformation

$$\begin{cases} x' = x \\ y' = y \\ z' = z \cdot f(x, y) \end{cases}$$

Transformed space and field



Designing devices for sub-wavelength imaging using a transformation-optics approach

- 1) Flat lenses designed by transformation-optics**
- 2) Reflection and transmission of flat lenses media**
- 3) Optical imaging devices based on flat lenses**

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